



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$\begin{aligned}
\therefore p &= 1 - \frac{4}{5\pi r^3} \left[\int_{\frac{1}{2}r}^r \int_0^\theta [15(\tfrac{1}{2}\pi - \theta)\cos\theta - 10\cos\theta + 10\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta - \tfrac{5}{2}\tan\tfrac{1}{2}\theta\sec^2\tfrac{1}{2}\theta] \times \right. \\
&\quad \left. x^2 dx d\theta + \int_0^{\frac{1}{2}r} \int_0^{\frac{1}{2}\pi} [15(\tfrac{1}{2}\pi - \theta)\cos\theta - 10\cos\theta + 10\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta - \tfrac{5}{2}\tan\tfrac{1}{2}\theta\sec^2\tfrac{1}{2}\theta] x^2 dx d\theta \right] \\
&= 1 - \frac{4}{5\pi r^3} \left[\int_{\frac{1}{2}r}^r \left(15(r-x)\cos^{-1}\left(\frac{r-x}{x}\right) - 10r - 20\sqrt{2rx-r^2} \right) \right. \\
&\quad \left. + \tfrac{5}{2}x - \frac{5x^2}{x + \sqrt{2rx-r^2}} \right) dx + \tfrac{1}{2} \int_0^{\frac{1}{2}r} x^2 dx \Big] = \frac{4}{\pi} (8\log 2 - 5).
\end{aligned}$$

Also solved with same result by the *PROPOSER*.

MISCELLANEOUS.

107. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

The index of refraction of a medium varying inversely as the square root of the distance, prove that the path of a ray of light in the medium is a cycloid.

Solution by the *PROPOSER*.

Taking the axis of y in the given plane and that of x at right angles to the y axis, letting $\mu = k/\sqrt{x}$ be the index of refraction, and $p = dy/dx$, we have, by the usual theory, for the differential equation to the path

$$\frac{dp/dx}{1+p^2} = \frac{1}{\mu} \left[\frac{d\mu}{dy} - \frac{d\mu}{dx} \frac{dy}{dx} \right] \dots (1). \quad \text{We have } \frac{d\mu}{dx} = -\frac{k}{2x^{\frac{3}{2}}} \dots (2),$$

$$\text{and (1) becomes } \frac{dp/dx}{1+p^2} = \frac{p}{2x}, \text{ or } \frac{dp}{p(1+p^2)} = \frac{dx}{2x} \dots (3).$$

$$\text{Integrating, } \log \frac{p}{\sqrt{1+p^2}} = \log \sqrt{x} + C \dots (4).$$

$$\text{Let } p=b, \text{ when } x=a; \text{ then } C = \log \frac{b}{a\sqrt{1+b^2}},$$

and (4) becomes $p = \frac{dy}{dx} = \frac{ax}{\sqrt{[(a^2/b^2)(1+b^2)x - x^2]}} \dots (5)$, the differential equation to a cycloid.

Also solved by G. B. M. ZERR, and L. C. WALKER.

108. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

To divide the arc of a cardioid into eight equal parts.